Bad Data Identification When Using Phasor Measurements

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Problem Definition

- Synchornized phasor measurement obtained by the phasor measurement are populating power systems.
- Traditional State estimation formulation needs to be modified when phasor measurements are present.
- This paper and related work are trying to propose such modification

Related Work

- "An Alternative for Including Phasor Measurement in State Estimators"
  - M.Zhou, V.Centeno, J.Thorp, A.Phadke
  - "*IEEE Tranactions On Power Systems, No.4 Nov, 2006*"
  - In this paper, authors provide the basic modification to apply phasor measurement to traditional state estimator.

Traditional State Estimator

A set of measurement [Z_1] consisting of non synchronized data of active and reactive power flows in network elements, bus injection, and voltage magnitudes at bus.
[E] is state vector, positive sequence voltages:
[ε] is the measurement error vector with a covarienace matrix [W_1]

$$[z_1] = [h_1(E)] + [\varepsilon_1]$$

Jacobian Matrix [H1] is obtained by taking partial derivatives of [h1] with respect to [E], such as:

$$[H_1(E)] = \left[\frac{\partial h_1(E)}{\partial (E)}\right]$$

Assuming a starting value for [E], one proceeds with iterations to obtain the weighted least-squares solution for state vector:

$$[E_{k+1}] = [E_k] + [G_1(E_k)]\left[H_1^T W_1^{-1}\right][z_1 - h_1(E_k)]$$

Where [G1(Ek)] is the gain matrix given I $[G_1(E_k)] = \left[H_1^T(E_k) W_1^{-1} H_1(E_k)\right]^{-1}$

Estimator with Phasor Measurements Mixed with Traditional Measurements

- Consisting of the vector [z1] and a set of positive sequence voltage and current phasors [z2] (the subscripts r and i representing the real and imaginary parts of the phasor measurements):

$$[z] = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \equiv \begin{bmatrix} z_1 \\ E_r \\ E_i \\ I_r \\ I_i \end{bmatrix}$$

- Voltage and Current Phasors are nonlinear functions of the state vector. Voltage phasor Ep at bus p and current phasor I(pq) are related to the bus voltages Ep and Eq. Where, the series admittance of the line connection buses p and q is

$$y_{(pq)} = \left(g_{(pq)} + jb_{(pq)}\right)$$

cont

- So accordingly:

$$[W] = \begin{bmatrix} W_1 & 0 \\ 0 & W_2' \end{bmatrix} \qquad [H(E)] = \begin{bmatrix} H_1(E) \\ H_2(E) \end{bmatrix} = \begin{bmatrix} \frac{\partial h_1(E)}{\partial (E)} \\ \frac{\partial h_2(E)}{\partial (E)} \end{bmatrix}$$

$$[E_{k+1}] = [E_k] + [G(E_k)] \left[H_1^T W_1^{-1}\right] [z_1 - h_1(E_k)] + [G(E_k)] \left[H_2^T W_2'^{-1}\right] [z_2 - h_2(E_k)]$$

$$[G(E_k)] = \left[H_1^T(E_k) W_1^{-1} H_1(E_k) + H_2^T(E_k) W_2'^{-1} H_2(E_k)\right]^{-1}$$

As Linear Problem

- **Converting to rectangular coordinates**: All angles are referred to the swing bus, whose angle is assumed to be zero. The rotation matrix [R'] is only for state vector rotation. So E(1), W1 can be easily rotated.

$$Cov([E])_{rect} = [R'][Cov([E])][R']^T \equiv [W_1'].$$

- In the rectangular coordinates, the combined measurement equation:

$$\begin{bmatrix} E_r^{(1)} \\ E_i^{(1)} \\ E_r \\ E_i \\ I_r \\ I_i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1' & 0 \\ 0 & 1' \\ C_1 & C_2 \\ C_3 & C_4 \end{bmatrix} \begin{bmatrix} E_r \\ E_i \end{bmatrix} \equiv [A] \begin{bmatrix} E_r \\ E_i \end{bmatrix}.$$

- 1 and 1' represents a unit matrix with 0 on diagonal. C1 through C4 are composed of line conductances and susceptances for those lines where current phasor measurements are avaiable.

$$\left[E^{(3)}\right] = [A^T W^{-1} A]^{-1} [W^{-1} A] [z']$$

- It can be proved that the two measurement equations are same.

Problems

- State estimation is formulated by choosing an arbitrary bus as the phase reference which is assumed to be zero. It create inconsistencies.
  So attacker can easily attack the reference bus, and cause whole estimation to be incorrect.
- Measurement Jacobian corresponding to the current phasors will become undefined when they are evaluated at flat start.

Jacobian

$$\frac{\partial I}{\partial V_n} = \frac{1}{V_n}\left(\frac{P}{S}\cdot\frac{\partial P}{\partial V_n} + \frac{Q}{S}\cdot\frac{\partial Q}{\partial V_n}\right) - \frac{S}{V_n^2}$$

$$\frac{\partial I}{\partial V_m} = \frac{1}{V_n}\left(\frac{P}{S}\cdot\frac{\partial P}{\partial V_m} + \frac{Q}{S}\cdot\frac{\partial Q}{\partial V_m}\right)$$

$$\frac{\partial I}{\partial \theta_n} = \frac{1}{V_n}\left(\frac{P}{S}\cdot\frac{\partial P}{\partial \theta_n} + \frac{Q}{S}\cdot\frac{\partial Q}{\partial \theta_n}\right)$$

$$\frac{\partial I}{\partial \theta_m} = \frac{1}{V_n}\left(\frac{P}{S}\cdot\frac{\partial P}{\partial \theta_m} + \frac{Q}{S}\cdot\frac{\partial Q}{\partial \theta_m}\right)$$

$$\frac{\partial \delta}{\partial V_n} = -\frac{1}{S^2}\left(P\cdot\frac{\partial Q}{\partial V_n} - Q\cdot\frac{\partial P}{\partial V_n}\right)$$

$$\frac{\partial \delta}{\partial V_m} = -\frac{1}{S^2}\left(P\cdot\frac{\partial Q}{\partial V_m} - Q\cdot\frac{\partial P}{\partial V_m}\right)$$

$$\frac{\partial \delta}{\partial \theta_n} = 1 - \frac{1}{S^2}\left(P\cdot\frac{\partial Q}{\partial \theta_n} - Q\cdot\frac{\partial P}{\partial \theta_n}\right)$$

$$\frac{\partial \delta}{\partial \theta_m} = -\frac{1}{S^2}\left(P\cdot\frac{\partial Q}{\partial \theta_m} - Q\cdot\frac{\partial P}{\partial \theta_m}\right)$$

S is the magnitude of complex power form bus n to bus m
P and Q are real and reactive power flow from bus n to m
V and θ are bus voltage magnitude and phase angle
I and σ are current magnitude and phase angle
So at flat start,, phse angles are assumed to be 0, so several Jacobian entries will be undefined at flat start.
So if the initialization is critical in state estimation, it brings errors.
So solution without reference initialization is needed.

Proposed Linear Rectangular Coordinates Hn

	E	F
P_i	$\frac{\partial P_i}{\partial E}$	$\frac{\partial P_i}{\partial F}$
Q_i	$\frac{\partial Q_i}{\partial E}$	$\frac{\partial Q_i}{\partial F}$
P_f	$\frac{\partial P_f}{\partial E}$	$\frac{\partial P_f}{\partial F}$
Q_f	$\frac{\partial Q_f}{\partial E}$	$\frac{\partial Q_f}{\partial F}$
E	1	---
F	---	1
C	$\frac{\partial C}{\partial E}$	$\frac{\partial C}{\partial F}$
D	$\frac{\partial D}{\partial E}$	$\frac{\partial D}{\partial F}$

P_i , Q_i , P_f , Q_f are real and reactive power injections and flows, respectively,
E and F are real and imaginary parts of bus voltages,
C and D are real and imaginary parts of currents.

Consider the real and reactive power flows from bus n to bus m:

$$P_{nm} = \frac{\left(E_n^2 + F_n^2\right)r - \left(E_n E_m + F_n F_m\right)r + \left(F_n E_m - E_n F_m\right)x}{r^2 + x^2}$$

$$Q_{nm} = \frac{\left(E_n^2 + F_n^2\right)x - \left(F_n E_m - E_n F_m\right)r - \left(E_n E_m + F_n F_m\right)x}{r^2 + x^2} - \left(E_n^2 + F_n^2\right)b$$

cont

$$\frac{\partial P}{\partial E_n} = \frac{2rE_n - rE_m - xF_m}{r^2 + x^2}$$

$$\frac{\partial P}{\partial E_m} = \frac{xF_n - rE_n}{r^2 + x^2}$$

$$\frac{\partial P}{\partial F_n} = \frac{2rF_n - rF_m + xE_m}{r^2 + x^2}$$

$$\frac{\partial P}{\partial F_m} = \frac{-rF_n - xE_n}{r^2 + x^2}$$

$$\frac{\partial Q}{\partial E_n} = \frac{2xE_n + rF_m - xE_m}{r^2 + x^2} - 2bE_n$$

$$\frac{\partial Q}{\partial E_m} = \frac{-rF_n - xE_n}{r^2 + x^2}$$

$$\frac{\partial Q}{\partial F_n} = \frac{2xF_n - rE_m - xF_m}{r^2 + x^2} - 2bF_n$$

$$\frac{\partial Q}{\partial F_m} = \frac{rE_n - xF_n}{r^2 + x^2}$$

r, x and b are resistance, reactance ans susceptance of n-m respectively.

$$C = \frac{(E_n - E_m)r + (F_n - F_m)x}{r^2 + x^2} - F_n b$$

$$D = \frac{(F_n - F_m)r - (E_n - E_m)x}{r^2 + x^2} + E_n b$$

$$\frac{\partial C}{\partial E_n} = \frac{r}{r^2 + x^2}$$

$$\frac{\partial C}{\partial E_m} = \frac{-r}{r^2 + x^2}$$

$$\frac{\partial C}{\partial F_n} = \frac{x}{r^2 + x^2} - b$$

$$\frac{\partial C}{\partial F_m} = \frac{-x}{r^2 + x^2}$$

$$\frac{\partial D}{\partial E_n} = b - \frac{x}{r^2 + x^2}$$

$$\frac{\partial D}{\partial E_m} = \frac{x}{r^2 + x^2}$$

$$\frac{\partial D}{\partial F_n} = \frac{r}{r^2 + x^2}$$

$$\frac{\partial D}{\partial F_m} = \frac{-r}{r^2 + x^2}$$

All these terms are well defined even at flat start, allowing calculation of the Jacobian and successful initialization of state estimation solution

cont

$$H_n \doteq \left[\begin{array}{c|c} H_{PF} & \\ H_{FF} & \\ H_{CF} & \\ \hline & H_{QE} \\ & H_{EE} \\ & H_{DE} \end{array}\right]$$

where:

$H_{PF} = \partial P / \partial F$, $H_{FF} = \partial F / \partial F$, $H_{CF} = \partial C / \partial F$

$H_{QE} = \partial Q / \partial E$, $H_{EE} = \partial E / \partial E$, $H_{DE} = \partial D / \partial E$

The gain matrix can then be written as:

$$G_n = \begin{bmatrix} G_F & 0 \\ 0 & G_E \end{bmatrix}$$

$$G_F = H_F^T W_F H_F \text{ and } G_E = H_E^T W_E H_E$$

$$H_F = \begin{bmatrix} H_{PF} \\ H_{FF} \\ H_{CF} \end{bmatrix} \text{ and } H_E = \begin{bmatrix} H_{QE} \\ H_{EE} \\ H_{DE} \end{bmatrix} .$$

Since no reference bus exists, the system will be declared as obserable if no zero pivots are encountered while factorizing G_F

Bad Data Processing

- Conventional network observability analysis will yield the number of observable islands in a given system.
- In order to be able to detect and identify errors in the phasor measurements, it needs two phasor measurements to ensure detectability and three for identification of bad data in a given observable island.

Simulation Results

- State estimation with and with a reference:

  Three PMUs are assumed to be exist at buses 5,18,37.

  Test A: Bus 5 as reference, and bus 5 has errors

  Test B: No reference any more

Test A	Test B	
Did not Converge	Measurement	Normalized Residual
	F5	53.99
	F18	44.59
	C18-4	34.24
	P15-45	16.19
	F37	15.85

cont

- Merging Observable Islands Using Phasor Measurement
  - Test A: Three Voltage Phasor Measurements
  - Test B: Four Voltage Phasor Measurements
  - Conclusion: Assign one phasor Measurement per island is sufficient to merge the islands into one network

Test A	Test B
Two Observable Islands:Island1 : Bus 4,5,6,17,19,20,21,22,23,24, 25,26 Island 2: Rest of Buses	Entire System is a Single Observable Island.

cont

Bad Data Identification in Phasor Measurement

Test A: Only One Voltage Phasor Measurement Bus 5 error

Test B: Three PMUs

Conclusion: Bus 5 error can be correctly detected

Test A		Test B	
Measurement	Normalized Residual	Measurement	Normalized Residuel
P5	0.044	F5	14.06
P4-5	0.036	F20	12.30
P5-6	0.035	P18-19	10.12
P11	0.015	P4-18	8.64
P41-43	0.015	E5	6.20

Reference

- "Bad Data Identification When Using Phasor Measurements", Jun Zhu and Ali Abur, *Proc. 2007 IEEE Lausanne Power TechPowerTech*, Lausanne, July 2007.
- "An Alternative for Including Phasor Measurement in State Estimators"

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Pros and Cons

- Pros:
  - This paper provides a solution without bus reference, so it is more *secure* solution compared with traditional method.
  - It discusses how to put their scheme into practice.
- Cons:
  - Extra Overhead, especially when the initialization state is not so important
  - Complexity in the computing

Discussion

- Securities issues from this paper:
  - PMU Security
  - Role of PMU played in State Estimation
  - Evasion (Traditional vs This paper)
  - Observability of the network